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A simple proof of the direct-style transformation

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January 2002

Abstract

We build on Danvy and Nielsen's first-order program transformation into continuation-passing style (CPS) to present a new correctness proof of the converse transformation, i.e., a one-pass transformation from CPS back to direct style. Previously published proofs were based on CPS transformations that were either higher-order, non-compositional, or operating in two passes, and were correspondingly complicated to reason about. In contrast, this work is based on a CPS transformation that is first-order, compositional, and that operates in one pass. Therefore the proof simply proceeds by structural induction on syntax.

Keywords: compositionality, CPS-transformation, direct-style transformation, correctness proof.

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1.1 The continuation-passing-style transformation

The CPS transformation on the λ-calculus maps direct-style expressions into semantically equivalent CPS expressions. Reynolds used it to map a functional program into an evaluation-order-independent form [9], and Plotkin later formalized it and proven it to be semantics preserving [8].

The CPS programs generated by Plotkin's CPS transformation contains so-called *administrative* redexes. Steele added a second pass to the transformation to reduce these redexes, generating equivalent but more compact CPS expressions [11]. This two-pass CPS transformation inspired several researchers to write *one-pass* CPS transformations that directly generate administratively reduced CPS expressions, making CPS transformation more practically useful. Appel, Danvy and Filinski, and Wand independently discovered a higher-order one-pass CPS transformation [1, 4, 12] while Sabry and Felleisen constructed a non-compositional one-pass CPS transformation based on syntactic theory [10]. Recently, however, Danvy and Nielsen presented a one-pass CPS transformation that is both first-order and compositional [6].

1.2 The direct-style transformation

The direct-style transformation is the inverse of the CPS transformation, and it maps CPS expressions back into direct-style expressions.

Danvy introduced the direct style transformation [2], and it was proven to preserve semantics by Lawall [7] and by Danvy, Dzafic, and Pfenning [3]. These proofs are based on the higher-order CPS transformation, and as such they require reasoning about higher-order functions, e.g., using logical relations.

This paper gives a simpler proof using only structural induction.

1.3 Overview

The rest of this paper is structured as follows: Section 2 presents the notation and definitions, Section 3 proves that the direct-style transformation preserves meaning by showing that it is a left inverse to the CPS transformation, and Section 4 concludes.

2 Definitions

A syntax for the λ-calculus is shown in Figure 1.

$$\begin{array}{rcll} p & ::= & e & p \in \text{DP}\textsc{rog} \\ e & ::= & t \mid s & e \in \text{DE}\textsc{xpr} \\ t & ::= & x \mid \lambda x.e & t \in \text{DT}\textsc{riv}\text{, trivial terms, i.e., values} \\ s & ::= & e\ e & s \in \text{DC}\textsc{omp}\text{, serious terms, i.e., computations} \\ & & & x \in \text{I}\textsc{de}\text{, a set of identifiers} \end{array}$$

Figure 1: Syntax of the λ-calculus in direct style

A grammar of λ-expressions in continuation-passing style is shown in Figure 2.

$$\begin{array}{rcll} p & ::= & \lambda k.e & p \in \text{CP}\textsc{rog} \\ e & ::= & r\ c \mid c\ t & e \in \text{CE}\textsc{xpr} \\ t & ::= & v \mid x \mid \lambda x.r & t \in \text{CT}\textsc{riv} \\ r & ::= & \lambda k.e \mid t\ t & r \in \text{CR}\textsc{oot} \\ c & ::= & k \mid \lambda v.e & c \in \text{CC}\textsc{ont} \end{array}$$

where $x \in$ IDE, $k \in$ CIDE, and $v \in$ VIDE, disjoint sets of identifiers.

Figure 2: Syntax of the λ-calculus in continuation-passing style

The call-by-value (CBV) CPS transformation is shown in Figure 3.

Being "fresh wrt. e" and "fresh wrt. c" means that we pick, deterministically, an element of CIDE, or two different elements of VIDE, that do not occur freely in e, respectively in c.

A typing argument shows that the CPS transformation actually generates only programs in continuation-passing style.

The direct-style transformation corresponding to the CPS transformation uses a stack of expressions to keep track of the intermediate results. This stack, represented by σ, is either the empty stack ($\bullet$) or a stack with something on top ($x :: \sigma'$), and the set of stacks of direct-style expressions is represented by [DEXPR].

The definition of the Direct-Style transformation is shown in Figure 4. The direct-style transformation is not total on the set of CPS programs. In

$$
\begin{array}{rcl}
\mathcal{C} & : & \textrm{DP}\textsc{rog} \rightarrow \textrm{CP}\textsc{rog} \\
\mathcal{C}[\![e]\!] & = & \lambda k.\,\mathcal{C}^{\textrm{DE}\textsc{xpr}}[\![e]\!]\ k \\
\\
\mathcal{C}^{\textrm{DE}\textsc{xpr}} & : & \textrm{DE}\textsc{xpr} \times \textrm{CI}\textsc{de} \rightarrow \textrm{CE}\textsc{xpr} \\
\mathcal{C}^{\textrm{DE}\textsc{xpr}}[\![t]\!]\ k & = & k\ \mathcal{C}^{\textrm{DT}\textsc{riv}}[\![t]\!] \\
\mathcal{C}^{\textrm{DE}\textsc{xpr}}[\![s]\!]\ k & = & \mathcal{C}^{\textrm{DC}\textsc{omp}}[\![s]\!]\ k \\
\\
\mathcal{C}^{\textrm{DT}\textsc{riv}} & : & \textrm{DT}\textsc{riv} \rightarrow \textrm{CT}\textsc{riv} \\
\mathcal{C}^{\textrm{DT}\textsc{riv}}[\![x]\!] & = & x \\
\mathcal{C}^{\textrm{DT}\textsc{riv}}[\![\lambda x.e]\!] & = & \lambda x.\lambda k.\,\mathcal{C}^{\textrm{DE}\textsc{xpr}}[\![e]\!]\ k \quad \text{where } k \text{ fresh wrt. } e \\
\\
\mathcal{C}^{\textrm{DC}\textsc{omp}} & : & \textrm{DC}\textsc{omp} \times \textrm{CC}\textsc{ont} \rightarrow \textrm{CE}\textsc{xpr} \\
\mathcal{C}^{\textrm{DC}\textsc{omp}}[\![t_1\ t_2]\!]\ c & = & \mathcal{C}^{\textrm{DT}\textsc{riv}}[\![t_1]\!]\ \mathcal{C}^{\textrm{DT}\textsc{riv}}[\![t_2]\!]\ c \\
\mathcal{C}^{\textrm{DC}\textsc{omp}}[\![s_1\ t_2]\!]\ c & = & \mathcal{C}^{\textrm{DC}\textsc{omp}}[\![s_1]\!]\ (\lambda v_1.v_1\ \mathcal{C}^{\textrm{DT}\textsc{riv}}[\![t_2]\!]\ c) \\
\mathcal{C}^{\textrm{DC}\textsc{omp}}[\![t_1\ s_2]\!]\ c & = & \mathcal{C}^{\textrm{DC}\textsc{omp}}[\![s_2]\!]\ (\lambda v_2.\mathcal{C}^{\textrm{DT}\textsc{riv}}[\![t_1]\!]\ v_2\ c) \\
\mathcal{C}^{\textrm{DC}\textsc{omp}}[\![s_1\ s_2]\!]\ c & = & \mathcal{C}^{\textrm{DC}\textsc{omp}}[\![s_1]\!]\ (\lambda v_1.\mathcal{C}^{\textrm{DC}\textsc{omp}}[\![s_2]\!]\ (\lambda v_2.v_1\ v_2\ c)) \\
 & & \text{where } v_1 \text{ and } v_2 \text{ are distinct and fresh wrt. } c
\end{array}
$$

Figure 3: Call-by-value CPS transformation

$$
\begin{array}{rcl}
\mathcal{D} & : & \textrm{CP}\textsc{rog} \rightarrow \textrm{DP}\textsc{rog} \\
\mathcal{D}[\![\lambda k.e]\!] & = & \mathcal{D}^{\textrm{CE}\textsc{xpr}}[\![e]\!]\ \bullet \\
\\
\mathcal{D}^{\textrm{CE}\textsc{xpr}} & : & \textrm{CE}\textsc{xpr} \times [\textrm{DE}\textsc{xpr}] \rightarrow \textrm{DE}\textsc{xpr} \\
\mathcal{D}^{\textrm{CE}\textsc{xpr}}[\![r\ c]\!]\ \sigma & = & \mathcal{D}^{\textrm{CC}\textsc{ont}}[\![c]\!]\ (\mathcal{D}^{\textrm{CR}\textsc{oot}}[\![r]\!]\ \sigma) \\
\mathcal{D}^{\textrm{CE}\textsc{xpr}}[\![c\ t]\!]\ \sigma & = & \mathcal{D}^{\textrm{CC}\textsc{ont}}[\![c]\!]\ (\mathcal{D}^{\textrm{CT}\textsc{riv}}[\![t]\!]\ \sigma) \\
\\
\mathcal{D}^{\textrm{CT}\textsc{riv}} & : & \textrm{CT}\textsc{riv} \times [\textrm{DE}\textsc{xpr}] \rightarrow \textrm{DE}\textsc{xpr} \times [\textrm{DE}\textsc{xpr}] \\
\mathcal{D}^{\textrm{CT}\textsc{riv}}[\![v]\!]\ (e :: \sigma) & = & (e, \sigma) \\
\mathcal{D}^{\textrm{CT}\textsc{riv}}[\![x]\!]\ \sigma & = & (x, \sigma) \\
\mathcal{D}^{\textrm{CT}\textsc{riv}}[\![\lambda x.r]\!]\ \sigma & = & (\lambda x.e, \sigma) \quad \text{where } (e, \bullet) = \mathcal{D}^{\textrm{CR}\textsc{oot}}[\![r]\!]\ \bullet \\
\\
\mathcal{D}^{\textrm{CR}\textsc{oot}} & : & \textrm{CR}\textsc{oot} \times [\textrm{DE}\textsc{xpr}] \rightarrow \textrm{DE}\textsc{xpr} \times [\textrm{DE}\textsc{xpr}] \\
\mathcal{D}^{\textrm{CR}\textsc{oot}}[\![\lambda k.e]\!]\ \sigma & = & (\mathcal{D}^{\textrm{CE}\textsc{xpr}}[\![e]\!]\ \bullet, \sigma) \\
\mathcal{D}^{\textrm{CR}\textsc{oot}}[\![t_1\ t_2]\!]\ \sigma & = & (e_1\ e_2, \sigma'') \quad \text{where} \quad (e_2, \sigma') \ = \ \mathcal{D}^{\textrm{CT}\textsc{riv}}[\![t_2]\!]\ \sigma \\
 & & \qquad\qquad\qquad\qquad (e_1, \sigma'') \ = \ \mathcal{D}^{\textrm{CC}\textsc{ont}}[\![t_1]\!]\ \sigma' \\
\mathcal{D}^{\textrm{CC}\textsc{ont}} & : & \textrm{CC}\textsc{ont} \times (\textrm{DE}\textsc{xpr} \times [\textrm{DE}\textsc{xpr}]) \rightarrow \textrm{DE}\textsc{xpr} \\
\mathcal{D}^{\textrm{CC}\textsc{ont}}[\![k]\!]\ (e, \sigma) & = & e \\
\mathcal{D}^{\textrm{CC}\textsc{ont}}[\![\lambda v.e]\!]\ (e', \sigma) & = & \mathcal{D}^{\textrm{CE}\textsc{xpr}}[\![e]\!]\ (e' :: \sigma)
\end{array}
$$

Figure 4: The direct-style transformation

the next section we show that it is total on the image of the CPS transformation, and we only consider the transformation on this set.

3 Correctness

We prove that the direct-style transformation is correct and non-trivial. By correct we mean that it preserves meaning. By non-trivial we mean that the direct-style expressions generated by the transformation are not only a limited subset of λ-expressions. Since CPS expressions are a subset of direct-style expressions, the identity function could be considered a trivial direct-style transformation.

The proof shows that the direct-style transformation is a left-inverse to the CPS transformation. Since the CPS transformation preserves meaning and is defined on all terms, the direct-style transformation must also preserve meaning and be non-trivial. The CPS transformation is injective but not surjective, so when restricted to its image, it is a bijection, and the left inverse also becomes a right inverse.

Lemma 1 (Left Inverse) *The $\mathcal{D}$ function is a left inverse to the $\mathcal{C}$ function.*

$$\forall p \in \mathrm{DP}\textsc{rog}.\mathcal{D}[\![\mathcal{C}[\![p]\!]]\!] = p$$

Proof:

The proof is by structural induction on the program. We show the following three properties by mutual structural induction.

1. If $e : \mathrm{DE}\textsc{xpr}$ is an expression and $k : \mathrm{CI}\textsc{de}$ a continuation identifier then for any σ
$$\mathcal{D}^{\mathrm{CE}\textsc{xpr}}[\![\mathcal{C}^{\mathrm{DE}\textsc{xpr}}[\![e]\!]\ k]\!]\ \sigma = e$$

2. If $t : \mathrm{DT}\textsc{riv}$ is a value then for any σ
$$\mathcal{D}^{\mathrm{CT}\textsc{riv}}[\![\mathcal{C}^{\mathrm{DT}\textsc{riv}}[\![t]\!]]\!]\ \sigma = (t, \sigma)$$

3. If $s : \mathrm{DC}\textsc{omp}$ is a computation and $c : \mathrm{CC}\textsc{ont}$ a continuation then for any σ
$$\mathcal{D}^{\mathrm{CE}\textsc{xpr}}[\![\mathcal{C}^{\mathrm{DC}\textsc{omp}}[\![s]\!]\ c]\!]\ \sigma = \mathcal{D}^{\mathrm{CC}\textsc{ont}}[\![c]\!]\ (s, \sigma)$$

Property 1: There are two cases, one for each production in the grammar.

Case $e = t$**:**

$$\begin{array}{ll} & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DExpr}}[\![t]\!]\ k]\!]\ \sigma \\ = & \mathcal{D}^{\text{CExpr}}[\![k\ \mathcal{C}^{\text{DTriv}}[\![t]\!]]\!]\ \sigma \\ = & \mathcal{D}^{\text{CCont}}[\![k]\!]\ (\mathcal{D}^{\text{CTriv}}[\![\mathcal{C}^{\text{DTriv}}[\![t]\!]]\!]\ \sigma) \\ = & \mathcal{D}^{\text{CCont}}[\![k]\!]\ (t, \sigma) \qquad\qquad \text{(by I.H.)} \\ = & t \end{array}$$

Case $e = s$**:**

$$\begin{array}{lll} \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DExpr}}[\![s]\!]\ k]\!]\ \sigma & = & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![s]\!]\ k]\!]\ \sigma \\ & = & \mathcal{D}^{\text{CCont}}[\![k]\!]\ (s, \sigma) \\ & = & s \end{array}$$

Property 2: There are two cases, one for each production in the grammar.

Case $t = x$**:**

$$\mathcal{D}^{\text{CTriv}}[\![\mathcal{C}^{\text{DTriv}}[\![x]\!]]\!]\ \sigma = \mathcal{D}^{\text{CTriv}}[\![x]\!]\ \sigma = (x, \sigma)$$

Case $t = \lambda x.e$**:**

$$\begin{array}{ll} & \mathcal{D}^{\text{CTriv}}[\![\mathcal{C}^{\text{DTriv}}[\![\lambda x.e]\!]]\!]\ \sigma \\ = & \mathcal{D}^{\text{CTriv}}[\![\lambda x.\lambda k.\,\mathcal{C}^{\text{DExpr}}[\![e]\!]\ k]\!]\ \sigma \\ = & (\lambda x.e', \sigma') \\ & \text{where } (e', \sigma') = \mathcal{D}^{\text{CRoot}}[\![\lambda k.\,\mathcal{C}^{\text{DExpr}}[\![e]\!]\ k]\!]\ \sigma \\ = & (\lambda x.e', \sigma') \\ & \text{where } (e', \sigma') = (\mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DExpr}}[\![e]\!]\ k]\!]\ \bullet\ , \sigma) \\ = & (\lambda x.e', \sigma') \\ & \text{where } (e', \sigma') = (e, \sigma) \qquad\qquad \text{(by I.H.)} \\ = & (\lambda x.e, \sigma) \end{array}$$

Property 3: There are four cases, one for each case of the $\mathcal{C}^{\text{DComp}}$ function.

Case $s = t_1\ t_2$**:**

$$\begin{array}{ll} & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![t_1\ t_2]\!]\ c]\!]\ \sigma \\ = & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DTriv}}[\![t_1]\!]\ \mathcal{C}^{\text{DTriv}}[\![t_2]\!]\ c]\!]\ \sigma \\ = & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (\mathcal{D}^{\text{CRoot}}[\![\mathcal{C}^{\text{DTriv}}[\![t_1]\!]\ \mathcal{C}^{\text{DTriv}}[\![t_2]\!]]\!]\ \sigma) \\ = & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\ & \text{where } (e_2, \sigma') = \mathcal{D}^{\text{CTriv}}[\![\mathcal{C}^{\text{DTriv}}[\![t_2]\!]]\!]\ \sigma \\ & \qquad\quad (e_1, \sigma'') = \mathcal{D}^{\text{CTriv}}[\![\mathcal{C}^{\text{DTriv}}[\![t_1]\!]]\!]\ \sigma \\ = & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (t_1\ t_2, \sigma) \end{array}$$

Case $s = s_1\ t_2$**:**

$$
\begin{array}{cl}
\multicolumn{2}{l}{\mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![s_1\ t_2]\!]\ c]\!]\ \sigma} \\
= & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![s_1]\!]\ (\lambda v_1.v_1\ \mathcal{C}^{\text{DTriv}}[\![t_2]\!]\ c)]\!]\ \sigma \\
= & \mathcal{D}^{\text{CCont}}[\![\lambda v_1.v_1\ \mathcal{C}^{\text{DTriv}}[\![t_2]\!]\ c]\!]\ (s_1, \sigma) \qquad \text{(by I.H.)} \\
= & \mathcal{D}^{\text{CExpr}}[\![v_1\ \mathcal{C}^{\text{DTriv}}[\![t_2]\!]\ c]\!]\ s_1 :: \sigma \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\
 & \text{where } (e_2, \sigma') = \mathcal{D}^{\text{CTriv}}[\![\mathcal{C}^{\text{DTriv}}[\![t_2]\!]]\!]\ s_1 :: \sigma \\
 & \qquad\quad (e_1, \sigma'') = \mathcal{D}^{\text{CTriv}}[\![v_1]\!]\ \sigma' \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\
 & \text{where } (e_2, \sigma') = (t_2, s_1 :: \sigma) \\
 & \qquad\quad (e_1, \sigma'') = \mathcal{D}^{\text{CTriv}}[\![v_1]\!]\ \sigma' \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\
 & \text{where } (e_2, \sigma') = (t_2, s_1 :: \sigma) \\
 & \qquad\quad (e_1, \sigma'') = (s_1, \sigma) \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (s_1\ t_2, \sigma)
\end{array}
$$

Case $s = t_1\ s_2$**:**

$$
\begin{array}{cl}
\multicolumn{2}{l}{\mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![t_1\ s_2]\!]\ c]\!]\ \sigma} \\
= & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![s_2]\!]\ (\lambda v_2.\mathcal{C}^{\text{DTriv}}[\![t_1]\!]\ v_2\ c)]\!]\ \sigma \\
= & \mathcal{D}^{\text{CCont}}[\![\lambda v_2.\mathcal{C}^{\text{DTriv}}[\![t_1]\!]\ v_2\ c]\!]\ (s_2, \sigma) \qquad \text{(by I.H.)} \\
= & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DTriv}}[\![t_1]\!]\ v_2\ c]\!]\ (s_2 :: \sigma) \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\
 & \text{where } (e_2, \sigma') = \mathcal{D}^{\text{CTriv}}[\![v_2]\!]\ (s_2 :: \sigma) \\
 & \qquad\quad (e_1, \sigma'') = \mathcal{D}^{\text{CTriv}}[\![\mathcal{C}^{\text{DTriv}}[\![t_1]\!]]\!]\ \sigma' \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\
 & \text{where } (e_2, \sigma') = (s_2, \sigma) \\
 & \qquad\quad (e_1, \sigma'') = (t_1, \sigma') \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (t_1\ s_2, \sigma)
\end{array}
$$

Case $s = s_1\ s_2$**:**

$$
\begin{array}{rl}
& \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![s_1\ s_2]\!]\ c]\!]\ \sigma \\
= & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![s_1]\!]\ (\lambda v_1.\mathcal{C}^{\text{DComp}}[\![s_2]\!]\ (\lambda v_2.v_1\ v_2\ c))]\!]\ \sigma \\
= & \mathcal{D}^{\text{CCont}}[\![\lambda v_1.\mathcal{C}^{\text{DComp}}[\![s_2]\!]\ (\lambda v_2.v_1\ v_2\ c)]\!]\ (s_1, \sigma) \qquad \text{(by I.H.)} \\
= & \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DComp}}[\![s_2]\!]\ (\lambda v_2.v_1\ v_2\ c)]\!]\ (s_1 :: \sigma) \\
= & \mathcal{D}^{\text{CCont}}[\![\lambda v_2.v_1\ v_2\ c]\!]\ (s_2, s_1 :: \sigma) \qquad \text{(by I.H.)} \\
= & \mathcal{D}^{\text{CExpr}}[\![v_1\ v_2\ c]\!]\ (s_2 :: s_1 :: \sigma) \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ \mathcal{D}^{\text{CRoot}}[\![v_1\ v_2]\!]\ (s_2 :: s_1 :: \sigma) \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\
& \text{where } (e_2, \sigma') = \mathcal{D}^{\text{CTriv}}[\![v_2]\!]\ (s_2 :: s_1 :: \sigma) \\
& \qquad\quad (e_1, \sigma'') = \mathcal{D}^{\text{CTriv}}[\![v_1]\!]\ \sigma' \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (e_1\ e_2, \sigma'') \\
& \text{where } (e_2, \sigma') = (s_2, s_1 :: \sigma) \\
& \qquad\quad (e_1, \sigma'') = (s_1, \sigma) \\
= & \mathcal{D}^{\text{CCont}}[\![c]\!]\ (s_1\ s_2, \sigma)
\end{array}
$$

These cases shows that the properties hold for all direct-style expressions, so in particular if $e : \text{DProg}$

$$\mathcal{D}[\![\mathcal{C}[\![e]\!]]\!] = \mathcal{D}[\![\lambda k.\,\mathcal{C}^{\text{DExpr}}[\![e]\!]\ k]\!] = \mathcal{D}^{\text{CExpr}}[\![\mathcal{C}^{\text{DExpr}}[\![e]\!]\ k]\!]\ \bullet = e$$

QED

When restricting the CPS transformation to its image, i.e., forcing it to be surjective, a left inverse is also a right inverse.

Lemma 2 (Right Inverse) *The function $\mathcal{D}$ is the right inverse of $\mathcal{C}$ on $\mathcal{C}[\![\text{DProg}]\!]$, the image of* DProg *under $\mathcal{C}$.*

$$\forall p \in \mathcal{C}[\![\text{DProg}]\!].\mathcal{C}[\![\mathcal{D}[\![p]\!]]\!] = p$$

Proof: Let $p \in \mathcal{C}[\![\text{DProg}]\!]$, i.e., there exists a $p' \in \text{DProg}$ such that $p = \mathcal{C}[\![p']\!]$. Then $(\mathcal{C}\circ\mathcal{D})(p) = (\mathcal{C}\circ\mathcal{D})(\mathcal{C}[\![p']\!]) = \mathcal{C}[\![(\mathcal{D}\circ\mathcal{C})(p')]\!]$. From Theorem 1 we know that $\mathcal{D}\circ\mathcal{C}$ is the identity on DProg, so $\mathcal{C}[\![(\mathcal{D}\circ\mathcal{C})(p')]\!] = \mathcal{C}[\![p']\!] = p$.

QED

With these lemmas showing the following connection

$$\begin{tikzcd} \text{DProg} \arrow[r, bend left, "\mathcal{C}"] & \mathcal{C}[\![\text{DProg}]\!] \arrow[l, bend left, "\mathcal{D}"] \end{tikzcd}$$

we can directly show correctness and non-triviality

Theorem 1 *The direct-style transformation is correct and non-trivial.*

Proof: Follows from the correctness of the CPS transformation and the previous lemmas. QED

4 Conclusion

We have presented a simpler proof of the correctness of the direct-style transformation than what has previously been published. The proof, like the previous ones, is based on a CPS transformation, since the choice of CPS transformation dictates the type of proof. Earlier proofs of the higher-order CPS transformation used logical relations [5], proofs of the non-compositional CPS-transformation used well-founded induction [10], and proofs of two-pass CPS transformations need to address both passes [10]. In contrast, a first-order, compositional, and one-pass CPS transformation allows a proof using only a single structural induction [6].

One can also show correctness of the direct-style transformation on larger sets than just the image of the CPS transformation. Proofs of such properties can also be derived from correctness of a CPS transformation, and using the first-order compositional CPS transformation also gives proofs using only structural induction.

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